Notes: End Behavior

I. End Behavior of Functions

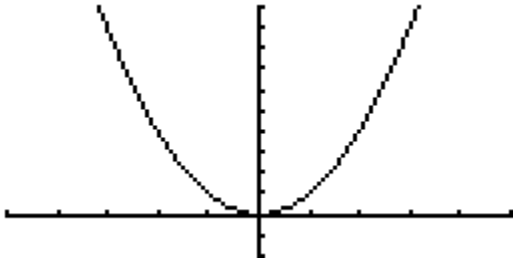
The end behavior of a graph describes the far left and the far right portions of the graph.

Using the leading coefficient and the degree of the polynomial, we can determine the end behaviors of the graph. This is often called the Leading Coefficient Test.

II. Leading Coefficient Test

END BEHAVIOR

$$f(x) = x^2$$



Degree: Even

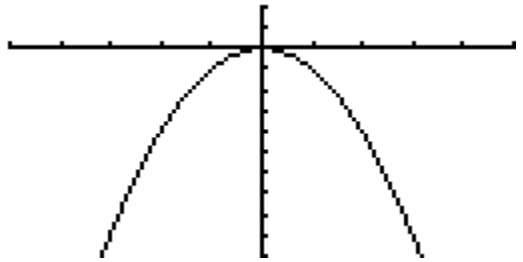
Leading Coefficient: +

End Behavior: Up Up ↖ ↗

$$y \to \infty \; as \; x \to -\infty, y \to \infty \; as \; x \to \infty$$

END BEHAVIOR

$$f(x) = -x^2$$



Degree: Even

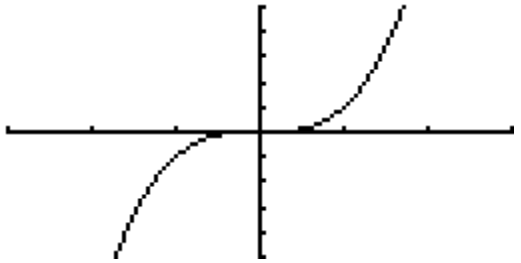
Leading Coefficient: −

End Behavior: Down Down ↙ ↘

$$y \to -\infty \; as \; x \to -\infty, y \to -\infty \; as \; x \to \infty$$

END BEHAVIOR

$$f(x) = x^3$$



Degree: Odd

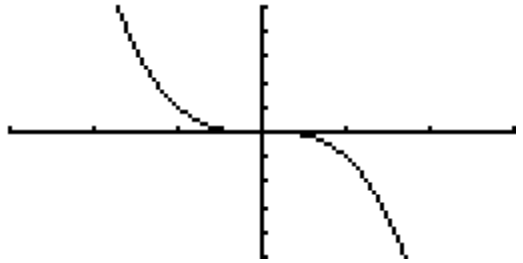
Leading Coefficient: +

End Behavior: Down Up

$$y \to -\infty \; as \; x \to -\infty, y \to \infty \; as \; x \to \infty$$

END BEHAVIOR

$$f(x) = -x^3$$



Degree: Odd

Leading Coefficient: −

End Behavior: Up Down

$$y \to \infty \; as \; x \to -\infty, y \to -\infty \; as \; x \to \infty$$

Examples: Describe the end behavior of the following function: $f(x) = 2x^2 + 3x - 5$

First determine whether the degree of the polynomial is even or odd.

degree = 2 so it is even

Next determine whether the leading coefficient is positive or negative.

Leading coefficient = 2 so it is positive

$$y \to \infty \; as \; x \to -\infty, y \to \infty \; as \; x \to \infty$$

PRACTICE: Describe the End Behavior:

a. $f(x) = -2x^3 + 5x - 9$

degree = 3 so it is odd

Leading coefficient = -2 so it is negative

$$y \to \infty \text{ as } x \to -\infty, y \to -\infty \text{ as } x \to \infty$$

b. $f(x) = 4x^4 - 2x^2 + 6x - 3$

degree = 4 so it is even

Leading coefficient = 4 so it is positive

$$y \to \infty \text{ as } x \to -\infty, y \to \infty \text{ as } x \to \infty$$

PRACTICE: Give the End Behavior:

c. $f(x) = 4x^5 - 3x^2 + 2x$

degree = 5 so it is odd

Leading coefficient = 4 so it is positive

$$y \to -\infty \text{ as } x \to -\infty, y \to \infty \text{ as } x \to \infty$$

d. $f(x) = -3x^4 + 2x^3 - x^2 + 3x - 4$

degree = 4 so it is even

Leading coefficient = -3 so it is negative

$$y \to -\infty \text{ as } x \to -\infty, y \to -\infty \text{ as } x \to \infty$$