Properties of Parallelograms

In this lesson and in the rest of the chapter you will study special quadrilaterals. A parallelogram is a quadrilateral with both pairs of opposite sides parallel. When you mark diagrams of quadrilaterals, use matching arrowheads to indicate which sides are parallel. For example, in the diagram at the right, \( PQ \parallel RS \) and \( QR \parallel SP \). The symbol \( \Box PQR \) is read “parallelogram \( PQRS \).”

**Theorems About Parallelograms**

**Theorem 6.2**
If a quadrilateral is a parallelogram, then its **opposite sides** are congruent.
\[ PQ \cong RS \text{ and } SP \cong QR \]

**Theorem 6.3**
If a quadrilateral is a parallelogram, then its **opposite angles** are congruent.
\[ \angle P \cong \angle R \text{ and } \angle Q \cong \angle S \]

**Theorem 6.4**
If a quadrilateral is a parallelogram, then its **consecutive angles** are supplementary.
\[ m\angle P + m\angle Q = 180^\circ, \text{ } m\angle Q + m\angle R = 180^\circ, \]
\[ m\angle R + m\angle S = 180^\circ, \text{ } m\angle S + m\angle P = 180^\circ \]

**Theorem 6.5**
If a quadrilateral is a parallelogram, then its diagonals bisect each other.
\[ QM \cong SM \text{ and } PM \cong RM \]

Theorem 6.2 is proved in Example 5. You are asked to prove Theorem 6.3, Theorem 6.4, and Theorem 6.5 in Exercises 38–44.
Using Properties of Parallelograms

FGHJ is a parallelogram. Find the unknown length. Explain your reasoning.

a. JH
b. JK

SOLUTION

a. \(JH = FG\)  
   \(JH = 5\)  
   Opposite sides of a \(\square\) are \(\cong\).
   Substitute 5 for \(FG\).

b. \(JK = GK\)  
   \(JK = 3\)  
   Diagonals of a \(\square\) bisect each other.
   Substitute 3 for \(GK\).

Using Properties of Parallelograms

PQRS is a parallelogram. Find the angle measure.

a. \(m\angle R\)
b. \(m\angle Q\)

SOLUTION

a. \(m\angle R = m\angle P\)  
   \(m\angle R = 70^\circ\)  
   Opposite angles of a \(\square\) are \(\cong\).  
   Substitute 70° for \(m\angle P\).

b. \(m\angle Q + m\angle P = 180^\circ\)  
   \(m\angle Q + 70^\circ = 180^\circ\)  
   Consecutive \(\triangle\) of a \(\square\) are supplementary.  
   Substitute 70° for \(m\angle P\).  
   Subtract 70° from each side.

Using Algebra with Parallelograms

PQRS is a parallelogram. Find the value of \(x\).

SOLUTION

\[m\angle S + m\angle R = 180^\circ\]  
\[3x + 120 = 180\]  
\[3x = 60\]  
\[x = 20\]  
Consecutive angles of a \(\square\) are supplementary.  
Substitute 3\(x\) for \(m\angle S\) and 120 for \(m\angle R\).  
Subtract 120 from each side.  
Divide each side by 3.
# REASONING ABOUT PARALLELOGRAMS

## EXAMPLE 4  Proving Facts about Parallelograms

**GIVEN**  $ABCD$ and $AEFG$ are parallelograms.

**PROVE**  $\angle 1 \equiv \angle 3$

**Plan**  Show that both angles are congruent to $\angle 2$. Then use the Transitive Property of Congruence.

**Solution**

**Method 1** Write a two-column proof.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $ABCD$ is a $\square$, $AEFG$ is a $\square$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 1 \equiv \angle 2$, $\angle 2 \equiv \angle 3$</td>
<td>2. Opposite angles of a $\square$ are $\equiv$.</td>
</tr>
<tr>
<td>3. $\angle 1 \equiv \angle 3$</td>
<td>3. Transitive Property of Congruence</td>
</tr>
</tbody>
</table>

**Method 2** Write a paragraph proof.

$ABCD$ is a parallelogram, so $\angle 1 \equiv \angle 2$ because opposite angles of a parallelogram are congruent. $AEFG$ is a parallelogram, so $\angle 2 \equiv \angle 3$. By the Transitive Property of Congruence, $\angle 1 \equiv \angle 3$.

## EXAMPLE 5  Proving Theorem 6.2

**GIVEN**  $ABCD$ is a parallelogram.

**PROVE**  $AB \equiv CD, AD \equiv CB$

**Solution**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $ABCD$ is a $\square$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Draw $BD$.</td>
<td>2. Through any two points there exists exactly one line.</td>
</tr>
<tr>
<td>3. $AB \parallel CD$, $AD \parallel CB$</td>
<td>3. Definition of parallelogram</td>
</tr>
<tr>
<td>4. $\angle ABD \equiv \angle CDB$, $\angle ADB \equiv \angle CBD$</td>
<td>4. Alternate Interior Angles Theorem</td>
</tr>
<tr>
<td>5. $DB \equiv DB$</td>
<td>5. Reflexive Property of Congruence</td>
</tr>
<tr>
<td>6. $\triangle ADB \equiv \triangle CBD$</td>
<td>6. ASA Congruence Postulate</td>
</tr>
<tr>
<td>7. $AB \equiv CD$, $AD \equiv CB$</td>
<td>7. Corresponding parts of $\equiv \triangle$ are $\equiv$.</td>
</tr>
</tbody>
</table>
**EXAMPLE 6 Using Parallelograms in Real Life**

**FURNITURE DESIGN** A drafting table is made so that the legs can be joined in different ways to change the slope of the drawing surface. In the arrangement below, the legs $AC$ and $BD$ do not bisect each other. Is $ABCD$ a parallelogram?

**SOLUTION**
No. If $ABCD$ were a parallelogram, then by Theorem 6.5 $AC$ would bisect $BD$ and $BD$ would bisect $AC$.

**GUIDED PRACTICE**

1. Write a definition of parallelogram.

2. Decide whether the figure is a parallelogram. If it is not, explain why not.

3. **IDENTIFYING CONGRUENT PARTS** Use the diagram of parallelogram $JKLM$ at the right. Complete the statement, and give a reason for your answer.

4. $JK \cong \_\_\_\_$

5. $MN \cong \_\_\_\_$

6. $\angle MLK \cong \_\_\_\_$

7. $\angle JKL \cong \_\_\_\_$

8. $\overline{JN} \cong \_\_\_\_$

9. $\overline{KL} \cong \_\_\_\_$

10. $\angle MNL \cong \_\_\_\_$

11. $\angle MKL \cong \_\_\_\_$

Find the measure in parallelogram $LMNQ$. Explain your reasoning.

12. $LM$

13. $LP$

14. $LQ$

15. $QP$

16. $m\angle LMN$

17. $m\angle NQL$

18. $m\angle MNQ$

19. $m\angle LMQ$
**Practice and Applications**

**Finding Measures** Find the measure in parallelogram $ABCD$. Explain your reasoning.

20. $DE$
21. $BA$
22. $BC$
23. $m\angle CDA$
24. $m\angle ABC$
25. $m\angle BCD$

**Using Algebra** Find the value of each variable in the parallelogram.

26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37.

38. **Proving Theorem 6.3** Copy and complete the proof of Theorem 6.3:

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

**Given** $ABCD$ is a $\square$.

**Prove** $\angle A \cong \angle C$,
$\angle B \cong \angle D$

**Paragraph Proof** Opposite sides of a parallelogram are congruent, so $a.$ and $b.$ By the Reflexive Property of Congruence, $c.$
$\triangle ABD \cong \triangle CDB$ because of the $d.$ Congruence Postulate. Because $e.$ parts of congruent triangles are congruent, $\angle A \cong \angle C$.

To prove that $\angle B \cong \angle D$, draw $f.$ and use the same reasoning.
39. **PROVING THEOREM 6.4** Copy and complete the two-column proof of Theorem 6.4: If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

**GIVEN** \(JKLM\) is a \(\square\).

**PROVE** \(\angle J\) and \(\angle K\) are supplementary.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (?)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (m\angle J = m\angle L, m\angle K = m\angle M)</td>
<td>2. (?)</td>
</tr>
<tr>
<td>3. (m\angle J + m\angle L + m\angle K + m\angle M = ?)</td>
<td>3. Sum of measures of int. (\triangle) of a quad. is 360°.</td>
</tr>
<tr>
<td>4. (m\angle J + m\angle J + m\angle K + m\angle K = 360°)</td>
<td>4. (?)</td>
</tr>
<tr>
<td>5. (2(? + ?) = 360°)</td>
<td>5. Distributive property</td>
</tr>
<tr>
<td>6. (m\angle J + m\angle K = 180°)</td>
<td>6. (?) prop. of equality</td>
</tr>
<tr>
<td>7. (\angle J) and (\angle K) are supplementary.</td>
<td>7. (?)</td>
</tr>
</tbody>
</table>

You can use the same reasoning to prove any other pair of consecutive angles in \(JKLM\) are supplementary.

**DEVELOPING COORDINATE PROOF** Copy and complete the coordinate proof of Theorem 6.5.

**GIVEN** \(PORS\) is a \(\square\).

**PROVE** \(\overline{PR}\) and \(\overline{OS}\) bisect each other.

**Plan for Proof** Find the coordinates of the midpoints of the diagonals of \(PORS\) and show that they are the same.

40. Point \(R\) is on the \(x\)-axis, and the length of \(\overline{OR}\) is \(c\) units. What are the coordinates of point \(R\)?

41. The length of \(\overline{PS}\) is also \(c\) units, and \(\overline{PS}\) is horizontal. What are the coordinates of point \(S\)?

42. What are the coordinates of the midpoint of \(\overline{PR}\)?

43. What are the coordinates of the midpoint of \(\overline{OS}\)?

44. **Writing** How do you know that \(\overline{PR}\) and \(\overline{OS}\) bisect each other?

**BAKING** In Exercises 45 and 46, use the following information.

In a recipe for baklava, the pastry should be cut into triangles that form congruent parallelograms, as shown. Write a paragraph proof to prove the statement.

45. \(\angle 3\) is supplementary to \(\angle 6\).

46. \(\angle 4\) is supplementary to \(\angle 5\).
**Stair Balusters**  In Exercises 47–50, use the following information.

In the diagram at the right, the slope of the handrail is equal to the slope of the stairs. The balusters (vertical posts) support the handrail.

47. Which angle in the red parallelogram is congruent to \( \angle 1 \)?

48. Which angles in the blue parallelogram are supplementary to \( \angle 6 \)?

49. Which postulate can be used to prove that \( \angle 1 \equiv \angle 5 \)?

50. **Writing** Is the red parallelogram congruent to the blue parallelogram? Explain your reasoning.

**Scissors Lift** Photographers can use scissors lifts for overhead shots, as shown at the left. The crossing beams of the lift form parallelograms that move together to raise and lower the platform. In Exercises 51–54, use the diagram of parallelogram \( ABCD \) at the right.

51. What is \( m\angle B \) when \( m\angle A = 120^\circ \)?

52. Suppose you decrease \( m\angle A \). What happens to \( m\angle B \)?

53. Suppose you decrease \( m\angle A \). What happens to \( AD \)?

54. Suppose you decrease \( m\angle A \). What happens to the overall height of the scissors lift?

**Two-Column Proof** Write a two-column proof.

55. **GIVEN** \( ABCD \) and \( CEFD \) are **s**.

   **PROVE** \( AB \equiv FE \)

56. **GIVEN** \( PQRS \) and \( TUVS \) are **s**.

   **PROVE** \( \angle 1 \equiv \angle 3 \)

57. **GIVEN** \( WXYZ \) is a **

   **PROVE** \( \triangle WMZ \equiv \triangle YMX \)

58. **GIVEN** \( ABCD, EBGF, HJKD \) are **s**.

   **PROVE** \( \angle 2 \equiv \angle 3 \)
59. **Writing** In the diagram, \(ABCG, CDEG,\) and \(AGEF\) are parallelograms. Copy the diagram and add as many other angle measures as you can. Then describe how you know the angle measures you added are correct.

60. **MULTIPLE CHOICE** In \(\square KLMN\), what is the value of \(s\)?

- **A** 5
- **B** 20
- **C** 40
- **D** 52
- **E** 70

61. **MULTIPLE CHOICE** In \(\square ABCD\), point \(E\) is the intersection of the diagonals. Which of the following is **not** necessarily true?

- **A** \(AB = CD\)
- **B** \(AC = BD\)
- **C** \(AE = CE\)
- **D** \(AD = BC\)
- **E** \(DE = BE\)

**Challenge**

**USING ALGEBRA** Suppose points \(A(1, 2), B(3, 6),\) and \(C(6, 4)\) are three vertices of a parallelogram.

62. Give the coordinates of a point that could be the fourth vertex. Sketch the parallelogram in a coordinate plane.

63. Explain how to check to make sure the figure you drew in Exercise 62 is a parallelogram.

64. How many different parallelograms can be formed using \(A, B,\) and \(C\) as vertices? Sketch each parallelogram and label the coordinates of the fourth vertex.

**MIXED REVIEW**

**USING ALGEBRA** Use the Distance Formula to find \(AB\). (Review 1.3 for 6.3)

65. \(A(2, 1), B(6, 9)\)

66. \(A(-4, 2), B(2, -1)\)

67. \(A(-8, -4), B(-1, -3)\)

**USING ALGEBRA** Find the slope of \(\overline{AB}\). (Review 3.6 for 6.3)

68. \(A(2, 1), B(6, 9)\)

69. \(A(-4, 2), B(2, -1)\)

70. \(A(-8, -4), B(-1, -3)\)

**PARKING CARS** In a parking lot, two guidelines are painted so that they are both perpendicular to the line along the curb. Are the guidelines parallel? Explain why or why not. (Review 3.5)

Name the shortest and longest sides of the triangle. Explain. (Review 5.5)

72. \(\triangle ABC\)

73. \(\triangle DEF\)

74. \(\triangle HJG\)